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Electromagnetic standing wave resonances in a periodically corrugated waveguide

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Mode interaction in a periodically corrugated waveguide is studied in detail. A resonance of non-Bragg type, an electromagnetic standing wave resonance, is predicted in the waveguide. The resonance is caused by the interaction of modes of different space harmonics. The resonance interaction results in spectrum splitting and in the appearance of forbidden gaps (stop bands). In this connection the waveguide spectrum takes miniband character with densely spaced stop bands. Different spectrum features change significantly the electromagnetic properties of the waveguide. [S1063-651X(98)50511-9]

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There are a lot of papers devoted to the investigation of electromagnetic wave propagation in periodic structures. The problem is topical in many fields of physics and technology: optics and integrated optics, lasers, holography, quantum electronics, acoustics, low-dimensional electron systems, and many others [1–6]. One would think that the problem has been well studied. However, more thorough analysis of the literature, which cannot be even partly cited in a rapid communication for a very big number of the works, shows that a resonance interaction of modes in periodic structures had not been investigated.

The problem was usually analyzed in the coupled-wave approximation [7,8]. The above method was applied to many cases of wave propagation in waveguide geometries with a periodic perturbation [2–4]. The coupled-wave equation gives solutions near the Bragg resonances [9,10], but it does not describe the interaction of modes of different space harmonics in a frequency range located far from the Bragg resonances. It happens because of initial exclusion of the interaction from the input equations.

In this Rapid Communication we show a solution of the problem in the multimode approximation that allows one to analyze this interaction and to reveal the electromagnetic standing wave resonance in a periodically corrugated waveguide.

Let us consider a corrugated waveguide made of a dielectric layer that occupies the space $d>y>y_0(x)$ and sandwiched between metal plates at $y=d$ and $y=y_0(x)$. The bottom plate has a periodic uneven shape defined by the function $y(x)=\xi\cos(qx)\equiv y_0(x)$, where $q=2\pi/a$, ξ and a are an amplitude and a period of the unevenness. In this case the problem reduces to solving the wave equation

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\varepsilon\,\frac{\omega^2}{c^2}\,\varphi=0 \tag{1}$$

with the boundary conditions

$$\varphi(x,y_0(x))=\varphi(x,d)=0, \tag{2}$$

where $\varphi(x,y)$ may be presented by, for example, E_z electric field component of transverse electric (TE) wave, ω is the wave frequency, and ε is the dielectric constant of the layer.

Due to the boundary periodicity, $\varphi(x,y)$ can represented in the form (Floquet's theorem)

$$\varphi(x,y)=\sum_n\,[a_n\cos(k_{y,n}y)+b_n\sin(k_{y,n}y)]\exp[\imath(k_x+nq_x)x], \tag{3}$$

where a_n and b_n are the Fourier coefficients, and $k_{y,n}$ and k_x are components of the wave vector $\mathbf{k}$.

Substituting Eq. (3) into the wave equation (1) gives the relation between ω and $\mathbf{k}$

$$\varepsilon\,\frac{\omega^2}{c^2}-(k_x+nq_x)^2-k_{y,n}^2=0, \tag{4}$$

and the boundary conditions (2) impose a relation on k_x and $k_{y,n}$ defining the allowed values $k_{y,n}$ and, hence, the dispersion law $\omega(\mathbf{k})$.

Substituting $\varphi(x,y_0(x))$ and $\varphi(x,d)$ into Eq. (2), we obtain the system of linear algebraic equations for coefficients a_n and b_n. The vanishing of the determinant of the system leads us to finding the allowed values $k_{y,n}$. In general it is impossible to solve the system analytically. However, in the case of a small amplitude of the irregularity, $\xi/d\ll 1$ and $\xi q\ll 1$, it is easy to obtain an analytical solution. It is not difficult to see that the coefficients a_n and b_n decrease $\sim\xi^{|n|}$ as n increases. That is why we can restrict ourselves to the approximation of three main space harmonics with the wave numbers k_0 and $k_{\pm 1}$.

Expanding Eq. (2) into a series with respect to the small parameters, we obtain the following characteristic equation, which determines the allowed values $k_{y,0}$

$$\tan(dk_0)=\frac{\xi^2}{4}k_0[k_{-1}\cot(dk_{-1})+k_{+1}\cot(dk_{+1})]. \tag{5}$$

In Eq. (5) and further, we omit the subsript y in the wave numbers $k_{y,n}$.

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Equation (5) is similar to the equation describing the phase relations for the electromagnetic wave obliquely incident on a thin dielectric layer [1].

We shall solve Eq. (5) by means of successive approximations about ξ [11]: $k_0=k_0^{(0)}+\delta k+\cdots$. From Eq. (5) if $\xi=0$ (a smooth waveguide) we have $\tan(k_0^{(0)}d)=0$; therefore,

$$k_0^{(0)}=\frac{p\pi}{d},\quad p=0,1,2,3,\ldots\ . \tag{6}$$

The solution (6) together with the dispersion equation (4) at $n=0$ define allowed modes in the smooth plane waveguide

$$k_x=\sqrt{\varepsilon\frac{\omega^2}{c^2}-\frac{p^2\pi^2}{d^2}}. \tag{7}$$

It is well known modes of a plane waveguide [2]. The cutoff frequency, ω_p, of each mode is defined as

$$\omega_p=\frac{c}{\sqrt{\varepsilon}}\frac{p\pi}{d}. \tag{8}$$

The next iteration gives the desired solution δk, which describes the effect of the boundary periodicity. In spite of its simple view the comprehensive solution of Eq. (5) takes a lot of space. In this Rapid Communication we will be concentrating on the analysis of resonance cases only, because of their importance for different applications.

In zero approximation the wave numbers $k_{\pm1}$ can be written as

$$k_{\pm1}=\sqrt{\frac{p^2\pi^2}{d^2}\mp2k_xq-q^2}. \tag{9}$$

From Eqs. (9) and (5) we see that at $k_x=\pm q/2$ the Bragg resonances appear as usual, due to an interference of waves propagating in the x direction. Moreover the condition of the Bragg resonance for different modes is unique and is the same as for an unbounded periodically inhomogeneous medium. It also has the same physical meaning as in the case of the unbounded medium.

Our goal is to analyze another type of resonance which is caused by the wave movement in the y direction. It follows from Eqs. (9) and (5) that the general resonance condition is

$$k_{\pm1}=\frac{l\pi}{d},\quad l=1,2,3,\ldots\ . \tag{10}$$

It is obvious that the formulas (10) and (6) include the condition of appearance of standing wave resonances, i.e., resonances between different modes of the $n=0$ and the neighboring $n=\pm1$ space harmonics. From Eq. (10) we may find a resonant value, $k_{x,p,l}^{\pm}$, of the wave number k_x at which the resonance between the pth mode of the $n=0$ space harmonic and the lth mode of the $n=+1$ or $n=-1$ harmonics take place correspondingly,

$$k_{x,p,l}^{\pm}=\mp\frac{q}{2}(1+\chi_{p,l}),\quad \chi_{p,l}=\frac{(l^2-p^2)\pi^2}{(dq)^2}. \tag{11}$$

The value of $\chi_{p,l}$ depends on the waveguide's parameters d and q, mode numbers p,l, and varies in a wide range of positive and negative values. In this connection, the resonance wave numbers $k_{x,p,l}^{\pm}$ may take values $k_{x,p,l}^{\pm}\to0$ at $\chi_{p,l}\to-1$, in contrast to the case of the Bragg resonances, when $k_x^{\pm}$ has the fixed values $k_x^{\pm}=\mp q/2$.

Combining Eqs. (11) and (4) yields the expression for the resonant frequency $\omega_{p,l}$,

$$\omega_{p,l}=[\omega_p^2+\omega_B^2(1+\chi_{p,l})^2]^{1/2}, \tag{12}$$

where $\omega_B=cq/(2\sqrt{\varepsilon})$ is the Bragg frequency.

It is easy to see from Eq. (12) that at $\chi_{p,l}\to-1$ the resonant frequency $(\omega_{p,l}-\omega_p)$, counted off from the cutoff frequency of the pth mode, vanishes. Thus, we come to the important conclusion that the electromagnetic standing wave resonances occur in a wide range of frequences starting with zero.

As one can see from Eq. (9), upon increasing k_x an alternation of the resonances between the pth mode of the $n=0$ and different modes of the $n=-1$ or $n=+1$ harmonics will occur. Here we consider only positive k_x because at $k_x<0$ the resonance condition remains the same; only the $n=-1$ harmonic must be replaced by the $n=+1$ harmonic. The resonances with the $n=+1$ space harmonic cut off at $k_x^c=(q/2)[(p\pi/dq)^2-1]$. Upon further increasing $k_x>k_x^c$ only the $n=-1$ harmonic remains resonant.

Near the resonances Eq. (5) reduces to the quadratic algebraic equation

$$\delta k^2-\frac{\xi^2}{4d^2}\left(\frac{l\pi}{d}\right)^2=0, \tag{13}$$

which gives two roots $\delta k(1,2)$,

$$\delta k(1,2)=\pm\frac{\xi}{2d}\frac{l\pi}{d}. \tag{14}$$

The solution (14) describes the spectrum splitting

$$\omega_{p,l}^{\pm}=\omega_{p,l}\pm\frac{\xi}{2d}\frac{l}{p}\frac{\omega_p^2}{\omega_{p,l}}, \tag{15}$$

and appearance of the stop bands $\delta\omega_{p,l}$

$$\delta\omega_{p,l}=\omega_{p,l}^{+}-\omega_{p,l}^{-}=\frac{\xi}{d}\frac{l}{p}\frac{\omega_p^2}{\omega_{p,l}}. \tag{16}$$

The width of the pass band $\Delta\omega_{p,l,l+1}$ between the lth and $(l+1)$th resonances in the spectrum of the pth mode may be derived from Eq. (12). It has a simpler view for the case of a thick layer, $\chi_{p,l}\ll1$,

$$\Delta\omega_{p,l,l+1}=\omega_{p,l+1}-\omega_{p,l}=\frac{(2l+1)\pi^2\omega_B^2}{(qd)^2\sqrt{\omega_B^2+\omega_p^2}}. \tag{17}$$

And, finally, in the case important for optic fibers, $\omega_B\gg\omega_p$, we get quite simple and suitable for experimental verification expressions for the pass and stop bands in the spectrum of the pth mode,

$$\Delta\omega_{p,l,l+1}=\frac{(2l+1)\pi^2\omega_B}{(qd)^2},\quad \delta\omega_{p,l}=\frac{\xi l\omega_p^2}{dp\omega_B}. \tag{18}$$

These equations show that the frequency spectrum of each waveguide's mode takes miniband character with densely spaced stop bands, since $\Delta\omega\ll\omega_B$. In connection with this, electromagnetic properties of the periodically corrugated waveguide will change in accordance with new features of the spectrum.

We would like to note that Eqs. (18) give the qualitatively right description of the transmission spectra [12,13] which were obtained, however, for optic fibers with the more complex periodic perturbation of its parameters and which were interpreted in different ways.

Thus, the derived theoretical investigation shows, while an electromagnetic wave is traveling in the layer with a periodically uneven boundary, that resonances between different modes of various space harmonics occur in the structure. The standing wave resonances cause the miniband behavior of the waveguide spectrum.

Here we have studied the problem solution in the first approximation. It is clear that we can consider the resonance interaction with the $n=\pm 2$ space harmonics in the next approximation. Then additional resonances will appear and they will lead to additional subdivision of the spectrum. More detailed analysis and applications to electron and acoustic systems will be given in a future publication.

It is worth noting that the phenomenon under consideration has some analogy with standing light wave interference [14,1]. However, for observation of the interference, one monochromatic wave is enough, while in our case the resonance appears as a result of the interference of two standing waves.

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